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A Markov decision sow model representing the productive lifespan of herd sows

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Abstract

A Markov decision sow model has been developed to represent the productive and reproductive lifespan of herd sows. This model precisely describes the herd structure at equilibrium based on actual farm data. Model outputs are the herd structure at equilibrium, and technical and economic indexes. Validation has been performed by comparing observed and simulated outputs from specific farm data. A complementary validation using a statistical χ^2 test based on Pearson's statistic is proposed to compare herd distributions at equilibrium. The model is intended to be used by farmers and runs on micro computers.

Keywords: Sow herds; Semi-Markov Decision Model; Modelling; Validation

1. Introduction

Today, decision-making in livestock production systems and in particular on pig farms is more difficult than in the past. Reasons for this include the intensification of production, the increase of competitiveness and the reduction of marginal profits. Rapid changes in market prices, production methods, biotechnology and communication systems contribute to increase the number of production strategies and the level of uncertainty. To maintain profitability, the farmer needs to estimate

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animal and economic responses to changing production conditions. In this way, decision-oriented models are increasingly needed and they are becoming essential in the process of decision-making.

Several mathematical models representing dairy cow and sow herd dynamics have been developed in the past focussing on reproduction and replacement decisions as reviewed by Jalvingh (1992, 1993) and by Kristensen (1993). Due to conceptual similarity in reproduction and replacement problems between dairy and swine farms, dairy models have often been modified to represent the sow herd dynamics (Dijkhuizen et al., 1986a,b; Huirne et al., 1988; Huirne, 1990; Jalvingh et al., 1992a; Jalvingh, 1993; Kristensen, 1993; Kristensen and Jorgensen, 1996). However, sow models are scarce in the literature probably because of the difficulty of representing the productive cycle of the sow which is faster and much subject to variation than the dairy cow cycle. In all cases, different mathematical methodologies have been used to develop these population models. Markov decision models are the most suitable to simulate herd dynamics since they can take into account within herd variation and represent the reproductive cycle of individual sows. Furthermore, they can be formulated either as probabilistic models to study different management strategies (e.g. Jalvingh et al., 1992a) or extended to Markov decision programming models (e.g. Huirne et al., 1988, 1993) to optimise the reproduction cycle.

The first published dynamic programming optimisation model for helping the culling of individual sows was developed by Huirne et al. (1988). Later, Jalvingh et al. (1992a) developed a swine herd model that evaluated the technical and economic consequences of tactical decisions concerning reproduction and replacement of sows. This work completed the previous one in order to estimate the consequences of management strategies, useful to find sub-optimum strategies. The model of Jalvingh et al. (1992a) was based on Markov chains designed to run on a weekly basis. However, this approach is not the most suitable when we are only interested in system behaviour in steady-state conditions. In general, all these models were validated subjectively by comparing simulated results to observed data. In few cases, models had been validated by comparing statistically simulated and observed data (Sorensen, 1990).

The objective of this project was to develop a semi-Markov decision sow model representing the productive lifespan of herd sows. Moreover, a new statistical validation methodology is proposed to evaluate the quality of the fit between simulated and observed data. The proposed model is aimed to help decision-makers in evaluating the animal and economic consequences of reproduction and replacement strategies taken by the farmer. For simplicity, only the stationary approach (steady state) is presented in this first paper.

2. Model design and implementation

Management of sow herds can be understood as a multistage process where farmers choose management rules to improve net returns over time. In the model, sows are production units that evolve through different productive states following

the same laws of motion. Sows are then characterised by their common productive behaviour and by the fact that they react equally to the same management strategies. Following these assumptions, the productive behaviour of the farm as a whole is the result of the aggregation of sows at the farm level. Thus, sows evolve through discrete intervals or stages (interval from conception to farrowing or abortion, interval from farrowing to weaning, interval from weaning to first mating, interval between matings, interval from abortion to mating, interval from mating to departure from the farm, etc.) where they can be in one of a finite number of states (gestation, lactation, etc.). It is at the end of any stage that the farmer can act to influence sow productivity in both current and future stages. The distribution of sows over states gives the herd structure from what total herd profit is then calculated by accumulating for each animal and for each state their economic performances. In this way, the impact of different management strategies can be analysed and compared.

Productive states are basically defined from the reproductive cycle and lifespan of sows. Thus, sows can be empty, gestating or lactating and each reproduction cycle is represented. Management strategies are those related to reproduction and replacement decisions and they are the maximal number of reproductive cycles and the maximal number of matings per cycle. Genetic and nutrition effects are not considered in this model directly. The model herein developed represents the productivity of a sow herd based on its population structure at equilibrium and some management strategies.

The model has been implemented in three main modules (Fig. 1). The first module calculates farm specific input parameters from farm data. These parameters are the probabilities that a sow goes from one productive state to another, the average sojourn time of sows in each state, mortality rates, culling rates and average litter sizes at birth and at weaning. Other input parameters not extracted from farm data are production costs, incomes and farm management strategies. Observed herd structure and several technical and economic indexes are also calculated from farm data for further verification. Farm specific parameters are used by the second module that contains the model itself. The model calculates the herd population structure at equilibrium in terms of number of sows per state and rate of sows moving from one state to another. The last module calculates several technical and economic indexes from the simulated population, compares these indexes to those calculated

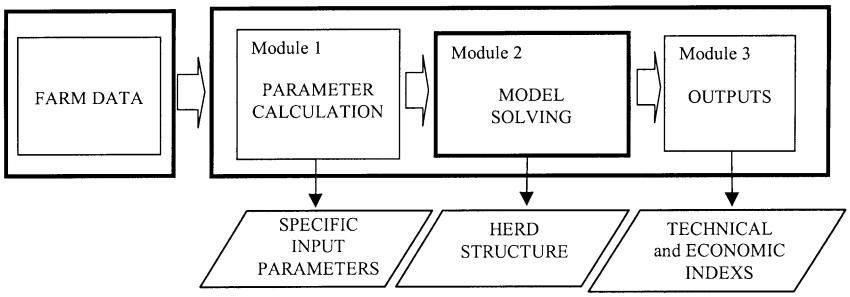

Fig. 1. General design of the model.

by the first module and finally, it compares statistically the observed and calculated population structures.

The model was developed in EXTEND™ (1997), an advanced tool for the development of models simulating decision-making processes. The model is available on PC under Windows or MacOS.

2.1. Model formulation

The model herein implemented has been developed based on the general Markov decision model presented by Plà et al. (1998). In this model, sows move from one state to another through transitions. Time interval between successive transitions is called a stage. Stages depend on states and they are of variable time-interval. Actions can be taken by farmers at the end of any stage in order to control the productive behaviour of the farm. Therefore, the represented multistage process is semi-Markovian. However, it is modelled as a standard Markovian process because only the herd in steady state situation is analysed (Howard, 1971; Puterman, 1994). The overall productive process has been represented into the model as a discrete and stationary Markov process homogenous in time.

Markovian decision models are characterised by the following elements: states S, actions A, transition probabilities and the reward function. The transitions of sows from one state $I \in S$ to another $j \in S$ are subjected to actions $a \in A$ that are chosen by farmers to modify the system behaviour. States and actions are considered finites. In an infinite planning horizon $\Omega = \{S \times A\}^{\infty}$ represents the set of all possible system paths (ω, i.e. all possible sequences of states and actions, $\omega = (i_1, a_1, i_2, a_2, \ldots, i_n, a_n, \ldots) \in \Omega$). The sequence of actions is the result of a policy or strategy, D.

For each policy D, $P^D = (p^D{}_{ij})$ is the transition matrix. In a Markovian-based model, future states S are defined as being only conditioned by the present state and not by the manner to which the present state is reached, that is,

$$P^D\{X_{n+1} = i_{n+1} | X_1 = i_1, Y_1 = a_1, \ldots, X_n = i_n, Y_n = a_n\} = P^D\{X_{n+1} = i_{n+1} | X_n = i_n, Y_n = a_n\}, \tag{1}$$

where $X_n(\omega) = i_n$ and $Y_n(\omega) = a_n$ are random variables that take values in S and A, respectively.

For any system path $\omega \in \Omega$, each action has some immediate economic effect and influences productivity in both current and future stages. Thus, decision-makers will establish a management strategy D according to their production objectives. The reward function, r, represents the farmer preferences in a decision theory context and it can be used in the building of a performance criterion as follows:

$$B_n^D(i) = r_i^D + \sum_{j \in S} p_{ij}^D B_{n-1}^D(j) \tag{2}$$

where the reward function r_i^D is the expected net return from a sow in the i-state and taking a-action determined by policy D, and $B_n^D(i)$ is the total return expected after n transitions from the initial state i. Expression (2) can be reformulated depending on different considerations and assumptions (e.g. total discounted expected returns or average expected returns per unit of time). Once a performance criterion is established, it is possible to analyse and compare different policies.

The sow reproductive cycle is represented in Fig. 2 which also illustrates sow's states and transitions. The productive life of a sow begins when entering the farm as a gilt. All gilts are purchased from outside the farm and they remain in a waiting state until first mating. Home-grown sows are not represented into the model since they are rare in commercial Spanish conditions. The productive life ends when sows are sold to the slaughterhouse or when they die.

In the model, sows can be culled depending on their reproductive condition and cycle or when the maximum number of matings or maximum number of cycles is achieved. Sold or dead sows can be replaced immediately or after some farm-specific delay. In this later case, sows stay in the fictive state "Purchasing new gilts" (Fig. 2). However, average replacement time is rarely known or may change over time. In the same way, sows may wait in the "Interval No mating to Sold" or "Interval Mating to Sold" states (Fig. 2). Model default values assumes that replacement and replaced

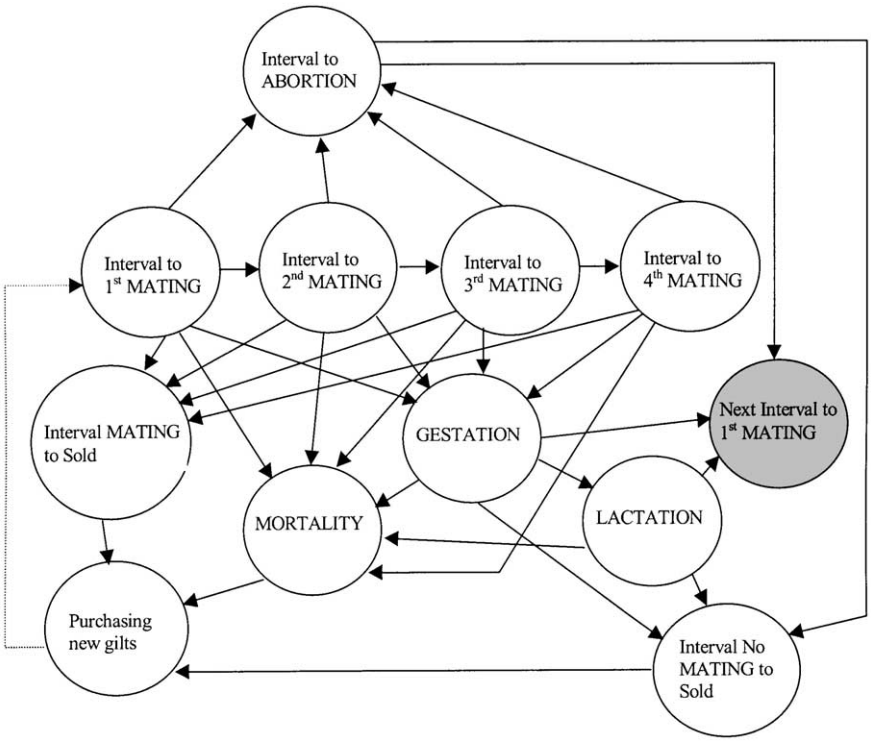

Fig. 2. States and transitions of a sow cycle.

sows have the same genetic potential and therefore, will have similar reproductive performances. The model simulates possible improvements on genetic merit in prolificacy throughout a rate of genetic improvement (Kristensen, 1988, 1993). Furthermore, the nutritional effects on reproduction are taken into account through reproductive parameters.

Farm data records rarely include heat or pregnancy detection information and therefore, the corresponding state of a sow is not known until a new mating or farrowing is recorded. Then, it is assumed that every sow that starts a stage in one state finishes the stage in the same state.

2.2. Estimation of population parameters

Under the assumptions made in this model, the dynamics of a herd under any given policy can be represented as a finite irreducible and aperiodic Markov chain. Transition probabilities are estimated from farm data during a fixed period of time. We assume that farm data represent farmer's management policy and therefore, no more transitions than observed are used in the estimation of probabilities. Maximum likelihood probabilities are calculated for each transition (Billingsley, 1961) as follows:

$$\hat{p}(j/i, a) = \frac{n_{ij}}{n_i} \tag{3}$$

where n_{ij} is the number of reproductive sows passing from i to j state, when action a is taken and $n_i = \sum_{k \in S} n_{ik}$ is the total number of reproductive sows that have passed throughout state i during the considered period. In this study, represented actions are those related to sow's replacement (i.e. keep and replace). The replace-action means that one sow is moved from states "Interval mating to sold", "Interval no mating to sold" or "Mortality" to "Purchasing new gilt" with probabilities equal to one.

Average time between transitions τ_i^D, that is, the expected time in days that sows spend in any state is also calculated. These values are used to estimate average sow's age over states, the reward function, feed consumption and production costs. Every state represented in the model (Fig. 2) has an average time interval associated to it. Main intervals are: gestation, lactation, weaning to mating interval, between matings interval, between last mating and replacement interval, etc.

2.3. Economic input parameters

Economic parameters are used in the model to define economic scenarios in order to calculate economic outputs and to compare economic results from different management strategies. This economic analysis is similar to the one used by other authors (Dijkhuizen et al, 1986b, Huirne et al., 1991, Jalvingh et al., 1992b, Kristensen, 1993).

Present net returns per state are calculated considering an annual discount rate of 6%. Total incomes result from both weaned piglets (33.06 €/piglet) and slaughter

sows (150.25 €/sow). Piglet and sow average prices are reported from the main local auction market on a per animal basis. Different costs are considered in the model. Fixed costs are estimated at 588.99 € per year for each sow in the farm. Variable costs are of 10.00 € per cycle. Other variable costs include gilts purchasing (150.25 €/gilt), artificial insemination (20.05 €/mating) and feeding cost. Feeding cost is calculated based on an average daily feed intake and feed type cost depending on reproductive condition (Table 1). Default values assumes no delay between purchasing and replacement of sows.

Parameters presented in Table 1 are used to define the reward value for each state, r_i^D, which represents the expected net return. Net return per state is defined from incomes and costs:

$$r_i^D = \beta_i \cdot (\mathrm{Ps}_i + \mathrm{Lw}_i \cdot \mathrm{Pw} - \mathrm{VC}_i - \mathrm{C}_i \cdot \mathrm{Pf}_i \cdot \tau_i - \mathrm{Pg}) \tag{4}$$

where

$\beta_i = e_i^{-q \cdot \tau_i}$	is the discount factor for state i, taking q as an annual interest rate;
Ps_i	is the value per sow sold to the slaughterhouse when i is one of the sold states. Otherwise is 0;
Lw_i	is the number of piglets weaned per litter when state i is a lactation state. Otherwise is 0;
Pw	is the value of weaned piglets;
VC_i	are variable costs per sow depending on state i. They include veterinary, artificial insemination and other expenses;
C_i	is the daily feed intake per sow in state i (kg);
Pf_i	is the feed price (€/kg);
τ_i	is the time interval of state i;
Pg	is the gilt value in the "purchasing gilt" state. Otherwise is 0.

Table 1
Average feed intake and feed costs

State	Feed cost (€/kg)	Feed intake (kg/day)
Open sows	0.13	2.0
Until first repetition	0.13	2.0
Until second repetition	0.13	2.0
Until third repetition	0.13	2.0
Gestation	0.13	2.5
Lactation	0.14	4.0
Until abortion	0.13	2.0
Piglets	0.24	0.2

2.4. Model outputs

Model outputs are calculated from both farm data and expected herd structure at equilibrium. These outputs are referred as observed or real and simulated or theoretical outputs, respectively. Model outputs are those related to the population distribution at steady-state conditions plus some technical and economical indexes (Fig. 3).

The observed structure of the population, $\Pi_o = (\pi^D_{11}, \pi^D_{12}, \ldots, \pi^D_{1|S|})$, is calculated from recorded farm data. Each component of the population structure π^D_{oi} is estimated by accumulating present number of sows in each state and by normalising the resulting vector. Observed technical indexes are then calculated following the specifications of the GTEP–IRTA® Spanish pig management information system (Noguera et al., 1992).

The expected herd distribution at equilibrium,

$$\Pi_s = \left(\pi^D_1, \pi^D_2, \ldots, \pi^D_{|S|}\right)$$

is calculated solving the following linear system of equations:

$$\begin{aligned} &\pi^D_j = \sum_{k \in S} \pi^D_k \hat{p}^D_{kj} \qquad j \in S \\ &\sum_{j \in S} \pi^D_j = 1 \end{aligned} \tag{5}$$

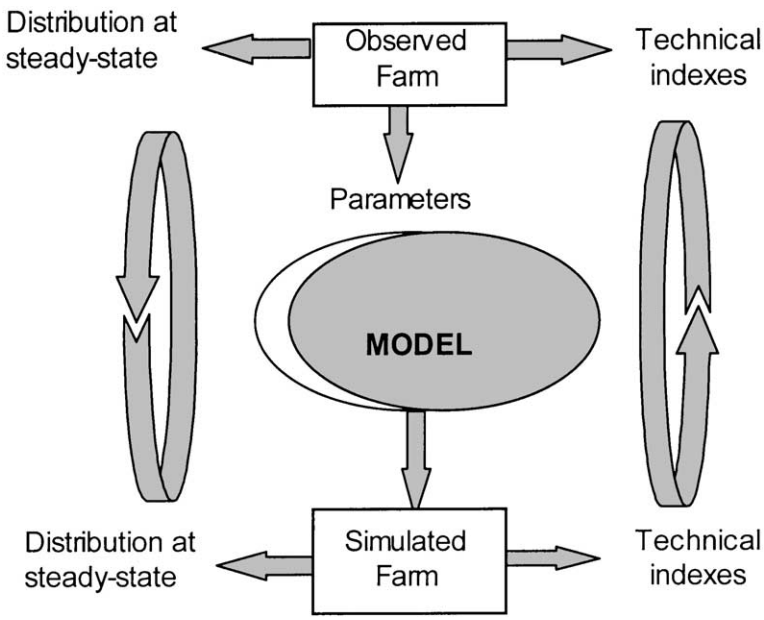

Fig. 3. Model validation process.

where $\hat{p}_{kj}^{D}$ represents the transition probability estimate for a sow to pass from k to j-state as calculated in (3), D is the policy derived from farm data and $\{\pi_j^D,\ j \in S\}$ represents the distribution at equilibrium when policy D is followed. From Eq. (5) and the reward function we can obtain the expected average net return per sow on farm:

$$B^D = \sum_{j \in S} \pi_j^D r_j^D \tag{6}$$

or the expected average net return per sow per day:

$$g^D = \sum_{j \in S} \frac{\pi_j^D r_j^D}{\pi_j^D \tau_j^D} \tag{7}$$

We should note that (6) and (7) are just two of the alternative formulations that can be derived from (2). Economic indexes like variable costs, feed cost, replacement cost, incomes from piglets and sows sold to slaughterhouses and gross margin are calculated as follows:

$$\begin{aligned}
&\mathrm{FC} = \mathrm{k}\sum_{j \in S} \beta_j \pi_j^D && \mathrm{VC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{VC}_j \\
&\mathrm{FeC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{C}_j \mathrm{Pf}_j \tau_j && \mathrm{RC} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{Pg} \\
&\mathrm{WI} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{Lw}_j \mathrm{Pw} && \mathrm{SI} = \sum_{j \in S} \beta_j \pi_j^D \mathrm{Ps}_j \\
&\mathrm{GM} = \mathrm{SI} + \mathrm{WI} - \mathrm{RC} - \mathrm{FeC} - \mathrm{VC} && \mathrm{NP} = \mathrm{GM} - \mathrm{FC}
\end{aligned} \tag{8}$$

where FC are fixed costs, VC are variable costs, FeC are feed costs, RC are replacement costs, WI are incomes from weaned piglets, SI are incomes from slaughter sows, GM is gross margin and NP is net profit. Furthermore, expected technical indexes are also calculated following the specifications of the same pig management information system referred to before (IRTA, 1996) but based on steady-state conditions. The expected average net profit as well as all other indexes are given in yearly basis. Finally, observed and expected population structures are compared statistically using Pearson's statistic (Billingsley, 1961).

3. Validation of the model

3.1. The evaluation process

Model verification was first performed by checking for both mathematical and logical consistencies. Thus, virtual examples were used to ensure the appropriateness of model calculations. Particular emphasis was given to the distribution of the population at equilibrium.

To evaluate the behaviour of the model in real conditions and the suitability of the steady-state approach, a random sample of ten farms was chosen from GTEP–IRTA®. Of these farms, three were discarded because of the inconsistency of their data (high number of missing data, etc.). Only 2 years of recorded farm data were considered for each farm. The first year data were used for estimating input parameters of the model and the second year data were used for validating the model results derived by using the first year data. The specific model input parameters were obtained as indicated in the previous section for the seven remaining farms (referred here as farms 1–7) from data collected during year 1996. Finally, each farm was simulated and the results compared to those observed in the database during the same 1997 period. Observed data and simulated results were compared graphically and numerically. Also, a non-parametric test (Pearson's statistic; χ^2-test) was used to check the goodness-of-fit. The desegregated values of χ^2-components were also provided and used to detect model components requiring further development. The following sections describe in detail the validation process outlined in Fig. 3.

3.2. Estimation of farm input parameters

All input parameters are farm specific and are calculated for each farm by the first module of the model (see Fig. 1). Input parameters from 1 year (1996) are used to simulate the following year's situation (1997) through expected herd distribution. For simplicity, only input parameters obtained for farm 4 are shown in this paper (see Tables 2, 3 and 4). Moreover, farm 4 was chosen because it was considered representative of the average farm in GTEP–IRTA® data-bank. Also, these parameters are assumed to be the result of farmer's management strategy and are therefore specific to each farm (e.g. lactation length). Table 2 gives averages and deviations of time intervals for each state. These time intervals are assumed to be independent of the reproduction cycle. For some of these parameters, differences between farms may be important, while within-farm variation is generally small. That is the case for the interval from last farrowing or weaning to market (Table 2) which shows important variation within farm and also between periods and farms (data not shown). We can also note that, in general, the interval from last mating to market is greater than the interval from last farrowing or weaning to market.

Table 2
Mean and standard deviation of time intervals for farm 4

	Mean (days)	S.D. (days)
Gestation length	114.8	1.3
Lactation length	29.6	2.1
Gestation with abortion length	90.0	0.0
Interval from last mating to market	39.9	0.5
Interval from last farrowing or weaning to market	16.9	1.9
Interval between matings	23.7	1.9
Age of purchased gilts	243.2	1.0

Table 3
Average and standard deviations of litter size, litter weaned and interval from weaning to first mating by reproductive cycle for farm 4

Cycle	LS[a]		LW[b]		IT1M[c]	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
1	9.37	1.81	8.47	1.65	1.00	0.00
2	10.69	1.49	9.14	1.48	9.21	2.48
3	11.42	1.79	8.97	1.47	6.97	1.31
4	11.05	1.65	8.95	1.54	7.14	2.10
5	11.43	1.86	8.78	1.59	7.00	2.09
6	10.45	1.78	9.05	1.24	6.24	1.24
7	10.87	1.80	8.47	1.54	6.40	1.36
8	10.73	1.86	9.27	0.87	7.73	2.72
9	9.82	1.50	4.64	1.88	9.18	2.82
10	7.40	1.98	6.20	2.27	7.60	1.40
11	11.33	1.30	7.00	2.26	7.33	1.12

[a] LS, litter size at farrowing.
[b] LW, litter size at weaning.
[c] IT1M, interval from weaning to first mating (days).

Sometimes, unusual values can be observed for the interval between matings. These values may result from reproduction or management problems and they should be understood as part of the real management strategy. Two of the less variable parameters were gestation length, biologically determined, and age of gilts. Other parameters such as gestation with abortion length or the interval between matings have standard deviations that depend on the number of observations.

Table 3 shows litter size, litter weaned and the interval from weaning to first mating per cycle in farm 4. Information from this and prior tables are used to estimate reward function values. In general, values from Tables 2 and 3 can be used directly as mean values (deterministic simulation) or they can be determined randomly taking into account the standard deviation (stochastic simulation) in order to reflect the variability-risk features of the problem (White, 1988).

Marginal probabilities calculated from farm 4 data are shown in Table 4. Marginal probabilities for conception rates are affected by the number of unsuccessful matings and by the reproduction cycle. Conception rates are expressed in relation to all matings per cycle. Abortion marginal probabilities are also specific for each reproduction cycle. Culling marginal probabilities are specific for each reproduction cycle and state. Because farmers rarely indicate the reason of sow replacement, culling rates in Table 4 also include casualties. Only conception rates for first and second matings are given in Table 4 because it was unusual to find farms with more than two matings per cycle. A 100% conception rate is given to parities 3 and up because farm data was inconsistent. Abortion rates are also low because they are rarely reported by farmers. Culling rates of 100% in the last cycle and state are given in order to end the sow lifespan.

Table 4
Marginal probabilities per reproductive cycle in farm 4

Cycle	Marginal probabilities[a]					
	CR(1)	CR(2)	AR	CRO	CRG	CRL
1	68.42	100.0	0.0	15.56	2.22	4.44
2	82.76	100.0	0.0	6.45	0.00	0.00
3	100.0	0.0	0.0	3.03	6.06	0.00
4	100.0	0.0	0.0	8.33	4.17	0.00
5	100.0	0.0	0.0	0.00	4.17	0.00
6	100.0	0.0	4.76	4.55	4.55	4.55
7	100.0	0.0	0.0	16.67	0.00	0.00
8	100.0	0.0	0.0	8.33	0.00	0.00
9	100.0	0.0	0.0	0.00	26.67	20.00
10	100.0	0.0	0.0	0.00	33.33	33.33
11	100.0	0.0	0.0	20.0	20.0	100.0

[a] CR: conception rates at first and second mating; AR, abortion rates; CRO, culling rates for open sows; CRG, culling rates for gestating sows; CRL, culling rates for lactation.

Some problems were sometimes found when calculating farm specific parameters by the first module. These problems were due to the lack of relevant data describing these parameters or to a low number of observations. To reduce the impact of such problems, in particular when estimating parameters in small farms, other estimation methods like empirical Bayes estimates instead of maximum likelihood estimates of transition probabilities could also be considered (Billiard and Meshkani, 1995).

3.3. Technical results and discussion

Results from each farm were obtained and evaluated. Farms were then classified according to differences between farm data and model predictions. For simplicity, only outputs from farms 1 and 4 are presented in this section. Farm 4 showed the best fit between observed and predicted population distribution parameters while farm 1 was the worst.

The main technical indexes (Table 5) obtained from simulated and observed farm data were in reasonable agreement in all farms. These indexes were selected from GTEP–IRTA® system and calculated in the same way. Observed productivity indexes are generally in agreement with simulated indexes. Normally, encountered differences are small, for instance the number of piglets born alive per litter is slightly different, in absolute terms, between simulated and observed indexes. This difference is explained by changes in the management policy, health conditions and herd structure, also it may explain the difference between the mortality rate of piglets until weaning. Reproductive rates are quite similar, although observed and simulated farrowing index (i.e. successful number of farrowings over number of matings) and repetition indexes are more different in farm 4 than in farm 1. Observed and predicted lactation lengths are almost equal because the farmers did

Table 5
Simulated and observed technical indexes for farms 1 and 4 (year 1997)

Indexes	Farm 1		Farm 4	
	Observed	Simulated	Observed	Simulated
Sows in farm (no.)	182	182	93	93
Piglets/sow/year (no.)	25.0	24.3	22.5	21.9
Litters weaned/Year (no.)	427	439	208	210
Abortions (%)	1.5	1.5	0.6	0.5
Born alive (no. piglets/litter)	11.47	10.44	11.50	10.41
Weaned (no. piglets/litter)	10.12	9.84	9.52	9.14
Mortality (%)	11.77	5.83	17.19	12.20
Farrowing/sow/year (no.)	2.47	2.47	2.37	2.40
Index of farrowing (%)[a]	0.89	0.86	0.74	0.79
Repetitions (%)	11.90	13.93	25.62	20.60
Interval weaning–oestrus (days)	5	6	7	7
Interval weaning–fertile mating (days)	6	6	11	8
Lactation length (days)	28	28	28	30
Interval between farrows (days)	148	148	154	152
Age at first farrowing (months)	12	12	12	12
Age at farrowing (months)	33	35	28	30
Age at replacement (months)	39	41	33	34
Litters/sow replaced (no.)	6.8	6.8	5.5	5.0
Sows replaced (%)	34.4	36.5	43.8	46.1

[a] Successful number of farrowings over number of matings.

not change the management policy to this respect and this variable is not affected by herd structure. Other sow intervals are however less constant because they are affected by herd structure showing sometimes discrepancies between simulated and observed values. Normally, the difference between simulated and observed weaning to fertile mating intervals should be the same as the difference between simulated and observed farrowing intervals because gestation and lactation lengths are close. In some farms (data not shown), discrepancies were detected in abortion rates, maybe because abortions are scarce and therefore distributions are unstable. Age of sows at first farrowing is the same in all farms because farmers use similar management strategies when entering gilts. Age of sows at farrowing is an index that provides an averaged age of sows in the farm. In both farms, this simulated age is higher than observed. In fact, the model applies casualty rates associated to each farm more uniformly. Differences reported in sow replacement (rate of sows replaced and age of replaced sows) are related to differences in litters per replaced sow as are observed. Analysis of these indexes shows that farm 1 has small differences between observed data and simulated results and also farm 4. Main differences were detected as shown in indexes affected by changes in management policy and the dynamics of the herd. Examples of these indexes are those reporting mating repetitions, abortions and replaced sows. Nevertheless the values were always rather similar or at least not enough different to discard the goodness of the model.

The number of sows per cycle and state for farms 1 and 4 are presented in Figs. 4 and 5. The open state includes sows waiting for insemination while the gestating state also includes sows which abort. Graphically, simulated results from farm 4 seem to be closer to observed data than those from farm 1. In this former farm 1, the number of sows in cycles 4–7 diverges strongly between observed and simulated data. This disagreement seems to be the result of a rapid increase in the number of gilts introduced in the herd a few years before data collection. In fact, the model represents the population distribution in steady-state conditions while farm data is an instant picture of the herd population which may not be at equilibrium. However, farm 4 seems to be in steady-state conditions as shown by the agreement between observed and simulated data (Fig. 5). Note that this situation was not appreciated so clearly in the analysis of technical indexes.

The χ^2-test was used to provide an objective measure of the goodness-of-fit between observed and simulated data. This test allows to compare the distribution (H_0:$\Pi_o = \Pi_s$) of the observed and simulated populations. The χ^2-test from all farms are presented in Table 6. Large values of χ^2 indicate that the null hypothesis is rather unlikely. The α-value gives a quantitative measure of the model goodness-of-fit. Low α-values indicate that discrepancies between observed and simulated data population are likely. Thus for farms 1, 2 and 6 the population distributions are different. These discrepancies were obvious after the visual analysis of farm 1, but not always was so clear. Hence the test plays a complementary role in the validation process, especially to discard bad fits. Discrepancies between observed and simulated data may originate from different causes. First, the model may not represent precisely the farm reality. Incorrect data from farms may also be responsible for the disagreement between simulated and collected data. However, it is important to note that a good model with appropriate farm data may also give different population distributions if external factors act. The degrees of freedom for the α-value calculation are related to productive states of the model and these states are specific to each farm.

The study of the χ^2 components can be used to overview the influence of each state and cycle on the total χ^2 value (Table 7). For example, the open sow state at the tenth cycle of farm 4 (see Table 7) represents 25% of the total χ^2 value (42.67 cf. Table 6). At this same state but at the second cycle, χ^2 component-values are also important. It is possible to appreciate these discrepancies in Fig. 5. Nonetheless, the analysis of the χ^2 components is much more powerful to appreciate differences that may exist between simulated and observed herd distribution data than the simple visual analysis.

3.4. Economic results and discussion

Economic results, including average expected profit, depend on both population structure derived from the model or from the farm and the economic scenario to be applied. Users can easily modify population and economic parameters defining the herd and the economic scenario to be simulated. An example of economic results produced by the model is shown in Table 8. Within farms, differences between simulated and observed farm results are generally in reasonable agreement with the

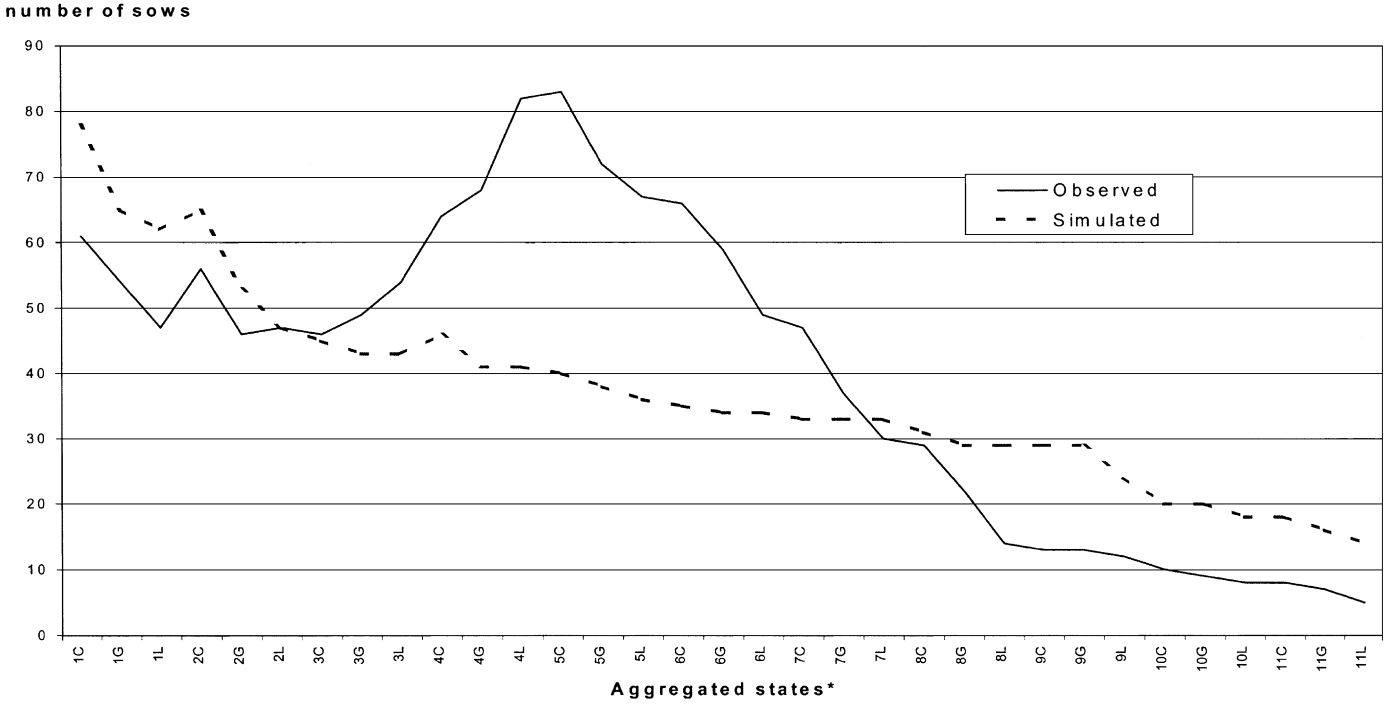

Fig. 4. Observed and simulated herd structure for farm 1 during year 1997.

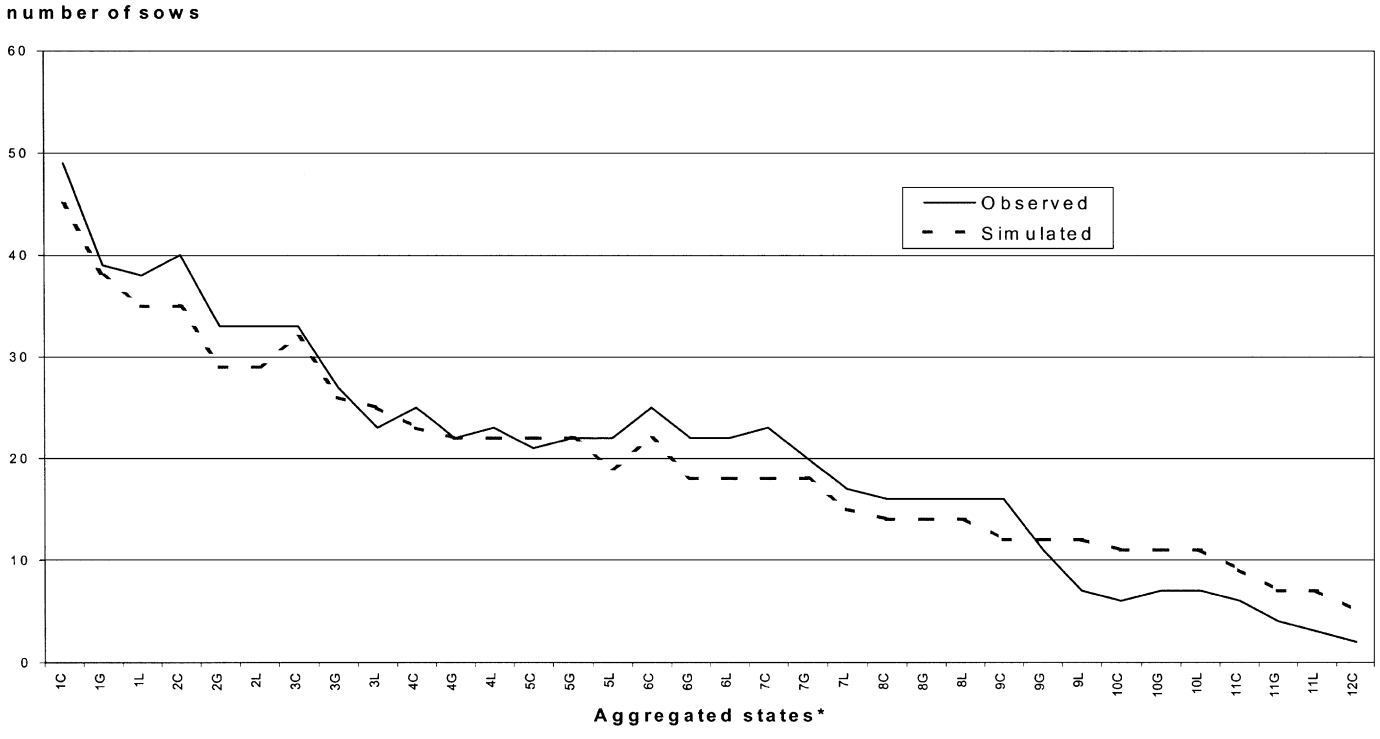

Fig. 5. Observed and simulated herd structure for farm 4 during year 1997.

Table 6
χ^2-test for limit distribution

Farm	No. sows	χ^2_{n-1}	n	α
1	182.4	386.22	67	0.000
2	162.0	173.64	60	0.000
3	49.9	43.23	50	0.740
4	92.9	42.67	55	0.867
5	138.2	35.80	57	0.861
6	220.1	373.71	38	0.000
7	251.0	49.11	47	0.731

exception of replacement costs and incomes from slaughtered sows. These differences between replacement costs and incomes from slaughtered sows are still important when compared between farms.

Farms with greater replacement costs have a greater slaughter value, and vice versa. The main income comes from weaned piglets and it is related to sow prolificacy, piglet mortality rate and reproduction efficiency. Variable and feed costs are quite close between farms. In farms 5 and 1, however, they are slightly higher, in farm 5 probably due to a shorter reproductive cycle and in farm 1 to a lower culling rate. Finally, farm 5 is the most and farms 4 and 6 are the least profitable farms with more than 90 euros per sow per year differences. These differences are explained by animal productivity (i.e. piglets weaned per sow per year) which is the main determinant of farm economic efficiency as shown in farms 5 and 4.

Replacement cost and slaughter income differences between observed and simulated results are determinant of the final net profit. Economic results show how the model, in general, can represent economical indexes which represent farm production

Table 7
Components of χ^2 value corresponding to each state in farm 4

Cycle	First mating	Second mating	Gestation	Lactation	Abortion	Mat–sold	FWA–sold
1	0.146	0	1.215	1.322	0	0.202	0.305
2	4.558	0.272	0.397	0.239	0.101	0.229	0.131
3	0.590	0.198	1.263	0.291	0	0	0.046
4	0.494	0	0.039	0.547	0	3.557	0.041
5	0.381	0	0.012	1.192	0	0.520	0
6	1.312	0.201	0.019	1.526	0	0.119	0.046
7	0.656	0.380	0.322	1.358	0	0.128	0.129
8	0.896	0	1.210	0.924	0	0.313	0.156
9	0.117	0	0.904	0.263	0	0.102	0.127
10	10.583	0	1.059	0.401	0	0	0.127
11	0.044	0	0.835	0	0	0	0.127

Mat–sold, stage corresponding from last mating to sold; FWA–sold, stage corresponding from farrowing, weaning or abortion to sold.

Table 8
Economic results expressed in € per sow per year (O: observed, S: simulated)

		Farm 1	Farm 2	Farm 3	Farm 4	Farm 5	Farm 6	Farm 7
AS	O	182.4	162.0	49.9	92.9	138.2	220.1	251.0
VC	S	43.80	41.15	43.12	42.33	44.10	41.76	42.04
	O	43.59	40.68	42.25	41.81	43.19	42.57	42.74
FeC	S	116.55	113.87	113.16	115.38	115.98	111.40	112.83
	O	116.56	113.41	114.24	114.89	117.01	110.98	113.02
RC	S	55.52	83.86	71.14	70.14	60.62	160.62	157.42
	O	95.71	178.12	197.46	142.44	114.77	87.34	71.98
WI	S	783.97	705.64	789.19	683.42	807.91	696.68	647.55
	O	782.57	686.97	825.54	671.51	850.75	684.45	653.16
SI	S	33.31	50.32	42.68	42.08	36.37	96.37	94.45
	O	57.42	106.87	118.48	85.46	68.86	52.40	43.19
GM	S	601.42	517.07	604.46	497.65	623.58	479.27	429.72
	O	584.15	461.63	590.06	457.83	644.64	495.96	468.61
NP	S	597.37	512.51	589.67	489.70	618.24	475.92	426.78
	O	580.10	457.07	575.27	449.88	639.30	492.61	465.67

AS, average of sows; VC, variable costs of sows; FeC, feed cost; RC, replacement cost; WI, piglets sold; SI, slaughter income; GM, gross margin; NP, net profit.

based on weaned piglets, but it frequently fails in indexes representing costs and incomes related with replacement.

Apart from comparing economic indexes within and among farms, it is also possible to explore other production alternatives from the animal or economical perspectives by performing sensitivity or post-equilibrium analyses. These analyses are essential for the farmer to gain insight related to the farm production process and to identify strategies that can improve the production efficiency and the expected net profit.

4. Concluding remarks

The proposed model is a semi-Markov decision model. It is shown that it can be used to represent specific swine farm situations. It has been formulated differently to other similar sow or cow Markov models already published (Dijkhuizen et al., 1986a; Huirne, 1990; Jalvingh, 1993; Kristensen and Jorgensen, 1996). The main difference between the proposed and previous model formulations is in the definition of states and stages, which in our model represent natural states and stages of the lifespan of the sow. Also, the model approach herein presented can be understood as

static because time is not driving model behaviour and is only concerned with steady state situations. This model structure was chosen to avoid splitting state variables into a greater number of states in order to improve the precision and speed of the model.

Former models estimate simulation parameters from previous published data assuming that these parameters can be applied to all farms. Model parameters in the proposed model are estimated from specific farm data to describe the animal and management characteristics of each simulated farm precisely. This characterisation of the farm allows the accurately representation of each farm and its validation by means of the comparison of farm results in terms of actual data and simulation results. Technical indexes were also introduced to improve model applicability because they are more familiar to farmers and technicians. However, it is difficult to objectively evaluate the resemblance between simulated results and observed values. This situation was also true when comparing results in previous sections and in previous published reports. Because of this difficulty, we have completed the validation process by testing the central hypothesis of the model with the χ^2 test. This statistical test provides a more objective measure of the model goodness-of-fit. Furthermore, the χ^2-components analysis helps to understand the model behaviour, especially when discrepancies between observed and simulated are not homogeneously distributed over states distributions.

Even with the statistical χ^2 test, it is not always easy to assert the suitability of the model for one specific farm. Exogenous variables like health problems, feeding changes, etc. may affect herd dynamics and make it difficult to explain differences that may appear between actual farm data and simulation results. In any case, this model is a powerful tool to study the components of the herd in steady-state situations, to answer questions of "What if...", etc. Because of the stationary approach, our model can be considered equivalent to others with static orientation in terms of expectations. Nevertheless, comparing expected results of herds at equilibrium is a good method of evaluating long term management alternatives as suggested by Jalvingh et al. (1992a,b). Furthermore, this model can be combined with optimisation techniques that can provide management strategies ensuring optimum farm performance. In fact, dynamic programming is one of such techniques with a recursive procedure to calculate optimal policies using Eq. (2).

In conclusion, the semi-Markov model herein presented precisely describes the herd structure at equilibrium based on actual farm data. In the future the model remains open for further improvements, for example, introducing optimisation procedures, making the model dynamic, etc.

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